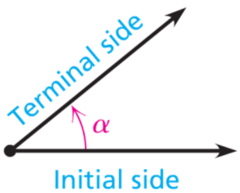
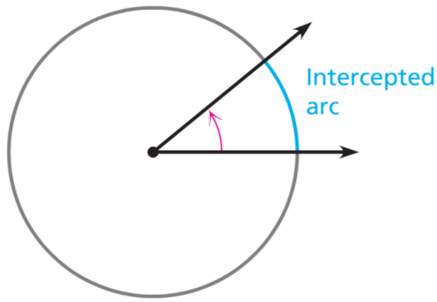
3.2 Circular Functions & Linear Velocity

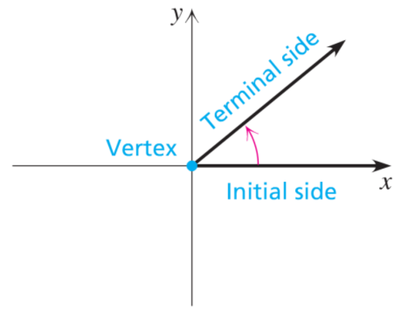
An **angle** can be formed by rotating one ray away from a fixed ray indicated by an arrow. The fixed ray is the **initial side** and the rotated ray is the **terminal side**. An angle whose vertex is the center of a circle is a **central angle**, and the arc of the circle through which the terminal side moves is the **intercepted arc**. An angle in **standard position** is located in a rectangular coordinate system with the vertex at the origin and the initial side on the positive *x*-axis.



Angle α



Central angle



Angle in standard position

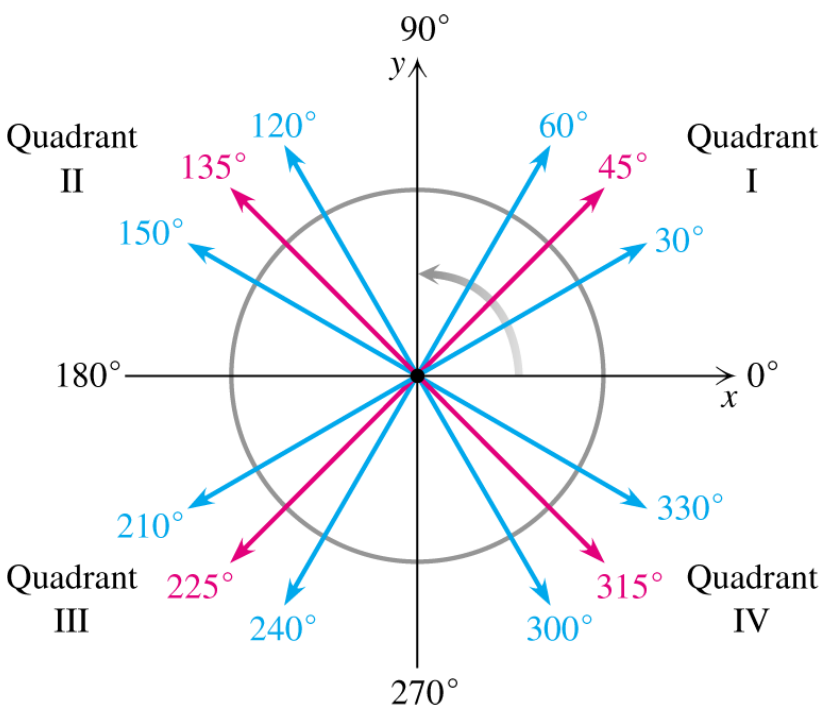
Degree Measure of Angles

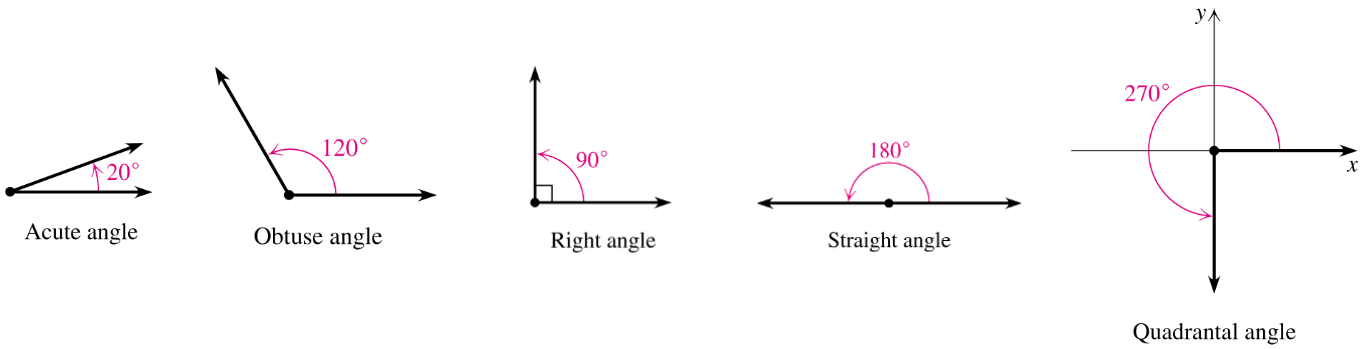
The measure, $m(\alpha)$, of an angle α is the amount of rotation from the initial side to the terminal side, and is found by using any circle centered at the vertex. An angle that forms a complete circle arc is 360°.

The **degree measure of an angle** is the number of degrees in the intercepted arc of a circle centered at the vertex.

Counterclockwise rotation—positive angle **Clockwise rotation—negative angle**

An angle in standard position is said to lie in the quadrant where its terminal side lies.





Acute angle—An angle with a measure between 0° and 90°.
Obtuse angle—An angle with a measure between 90° and 180°.
Straight angle—An angle with a measure of exactly 180°.
Right angle—An angle with a measure of exactly 90°.
Quadrantal angle—An angle in standard position whose terminal side is on an axis.

The terminal side of an angle may be rotated in either a positive or negative direction to get to its final position. It can also be rotated for more than one revolution in either direction. Two angles with the same terminal side are called **coterminal angles.**

Coterminal Angles—Angles α and β are coterminal if and only if there is an integer k such that $m(\beta) = m(\alpha) + k360°$. To find coterminal angles in degrees, add and subtract multiples of 360°.

Examples: Find two positive angles and two negative angles that are coterminal with each angle:
a) 23°
b) –146°

Examples: Determine whether the angles in each pair are coterminal:
a) –128° and 592°
b) 8° and –368°

Example: Draw each angle in standard position, then name the quadrant in which the terminal side lies.
a) 255°
b) –650°
c) 1360°

Coterminal Angles:
To find coterminal angles in radians, add or subtract multiples of 2π. Make sure to find a common denominator.

Examples: Find two positive angles and two negative angles that are coterminal with the given angle.

a) $\frac{5\pi}{6}$ b) $-\frac{\pi}{4}$ c) $\frac{7\pi}{3}$ d) 1.4

Velocity: The rate at which the location of an object is changing with respect to time.

Angular Velocity: The rate at which the angle is changing. If a point is in motion on a circle through an angle of α radians in time t, then its angular velocity ω is given by $\omega = \frac{\alpha}{t}$. Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

Examples:
Convert 650 rpm (revolutions per minute) to radians per minute.
(Use the fact that $1 \text{ revolution} = 2\pi \text{ radians}$).

Convert the angular velocity of 1600 rad/hr to rad/sec.

A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

Find the angular velocity in radians per second for a particle that is moving in a circular path at 4 revolutions per second on a circle of radius 9 ft.

Linear Velocity: The rate at which the distance is changing. If a point is in motion on a circle of radius r through an angle of α radians in time t, then its linear velocity v is given by $v = \frac{s}{t}$, where s is the arc length determined by $s = \alpha r$.

Examples:
A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per minute for a point on the tip of the propeller?

Find the linear velocity in meters per second for a particle that is moving in a circular path at 7 revolutions per second on a circle of radius 15 meters.

What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpms?

Find the linear velocity in miles per hour for a particle that is moving in a circular path at 1800 revolutions per minute on a circle with a diameter of 14 inches.

Linear Velocity in Terms of Angular Velocity: If v is the linear velocity of a point on a circle of radius r, and ω is its angular velocity, then $v = r\omega$.

Example:
Any point on the surface of the earth (except at the poles) makes one revolution (2π radians) about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is $2\pi/24$ or $\pi/12$ radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).